Drop Fingering on Oblique Impact: Part 2—Modeling

Alexander L. Brown* and Richard A. Jepsen
Sandia National Laboratories
PO Box 5800
Albuquerque, NM 87185-1135

Abstract

Recently acquired data for oblique drop impacts provides the background for the development of a model to help describe the fluid behavior of such events. Part 1 of this paper series presents data from impact events of various scales with water. Fingers are particularly evident in portions of the results, and have therefore been quantified for the range of experimental conditions.

The fingering data have been analyzed to develop a predictive model that can appropriately distribute the fingers given the initial direction of travel of the incoming drop and the angle of inclination of the drop to the obstacle surface. This is managed through an empirical distribution function. The distribution function has been empirically determined from the dimensionless data, and is a function of two constants and a dimensionless velocity. The model is found to result in reasonable fit for the range of conditions that involve between 0.4 and 10 cm diameter drops, velocities ranging from about 1-20 m/s, and impact angles from 90º to 45º (from horizontal). Coefficients of determination are presented for the existing comparison. Because the distribution function only provides a weighting, we also make comparisons with the data involving a total finger model. The fit remains acceptable with the added uncertainty of the total finger model.

This model provides the basis for an oblique impact splashing model that has been implemented in a dilute spray CFD code. We make the assumption that the splashing drops are distributed proportionally according to the fingering distribution. This is an excellent approximation for some splashes, and may be reasonable for others. This model represents a pragmatic improvement to the predictive behavior of a general model for drop impacts.

Introduction

We are primarily interested in high consequence impact and fire events. These include transportation accidents involving tanks of liquid fuels and the velocities or circumstances at or beyond which those tanks will rupture and result in a significant fire. These events are complex, and require cross-disciplinary expertise to begin to understand how to model and approximate the results from the event. The events include multi-material multi-phase deformation and impact dynamics. Complex liquid spread and splashing occurs, and influences the nature of the subsequent fireball and surface liquid fire. Very little test data exist at this scale, which is one of the challenges to model development. A secondary interest is fire suppression sprays.

A subset problem relating to the principal goals includes the ability to model a single drop impacting on a dry surface or a liquid. Two excellent reviews cover much of the historical research in this regard [1,2]. A recent review of existing drop impact models by Cossali et al. serves to highlight many of the shortcomings of the empirical drop impact models in the open literature [3]. Most existing models are only appropriate in limited regimes of velocity and drop diameter, and much of the fidelity that might be expected from such a model is not reproduced in the models. Worse, many of the models result in widely divergent predictions. Some of them are obviously incorrect at extremes, and do not give realistic results unless they are in narrow regimes. None of the models described provide details on the directional mass distribution of the splash, or the fraction of mass remaining on the surface following an impact. Most historical splashing models are designed for combustion spray applications, which may help justify some of the limitations. Our application of principal interest is in a different regime, but should be capable of being predicted by a common model.

* Corresponding author, albrown@sandia.gov

Some improvements have been made towards resolving issues for our application since the Cossali et al. [3] work was published. Large-scale testing has provided the data necessary to construct a model to predict the mass involved in the splash appropriate for water drops. A new finger model has also been proposed that better fits these drop impacts that involve high velocity and large diameter [4]. A new recognition of the significance of the ambient medium has been demonstrated in the impact data of Xu et al. [5,6] for various pressures. We also recently presented a model to determine splash mass from an impact [7]. Recent data in the paper titled Part 1 of this work provides evidence for drop impacts across a range of angles and for many other relevant conditions known to affect the outcome of an impact. Because the evolution of fingers is the most obvious expression of the variations in the data, the finger counts at various angular regimes have been measured for model development.

Employing all of the information available, we have designed a model that can predict the finger results of an oblique impact on a dry surface. The fingering is predicted on the basis of a weighting function, and the integral of the weighting function can be employed to allocate fingers in a model as is suggested by a fit to the data. The model and its fit are presented.

Methods

Finger counts for a wide variety of tests are found in Part 1 of this paper series. Conditions varied in the test series include fluid type, angle of impact, drop velocity, drop diameter, pressure, surface roughness, and drop size. But analysis is limited to variations in angle of impact, drop velocity, and drop diameter for this analysis because analyzed data from the other variations are insufficient to incorporate in model development at this point. This study includes water impacts with velocities from 1-20 m/s, diameters from 0.002 to 0.1 m, and angles from 90 to 40 degrees from vertical. Pressures included are that of natural atmospheric variations between the two test laboratories. Fingering is defined as the early expression of instabilities at the fringes of the spreading fluid. In some instances, fingers merge as time progresses, and the later finger count has not been studied as carefully.

Figure 1 shows graphically the nomenclature for the number of fingers counted in the experimental tests. Because we have observed that the distribution of fingers is a relatively smooth function with variations in each quadrant, we believe that we can adequately reproduce the fingering distribution by evaluating the finger counts in the hemispheres (HU, HD) and in the quarters (QU, QD, QM) and a smooth function that describes that variation.

The relations for the impact dynamics are all presented from the reference plane of the impacted surface. Figure 2 shows many of the variables with a graphical representation of their meaning. We propose a form for the distribution function of the fingers:

$$f(\psi, r) = 1 - f_1(r)\sin^2\psi - f_2(r)\sin^2\frac{\psi}{2} \tag{1}$$

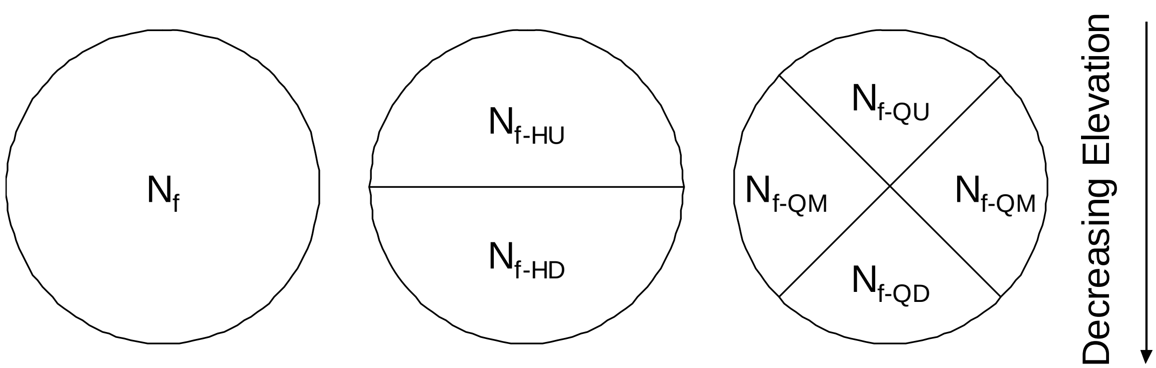

Figure 1. An illustration of the finger numbering nomenclature for a vertical drop.

The form of this function was inspired by the form of a basic scattering phase function for radiation interacting with a small sphere. The two free functions are proposed to be simple linear functions of the velocity ratio:

$$f_1(r) = Ar \tag{2}$$

$$f_2(r) = Br \tag{3}$$

The variations due to incoming attack incidence angle are managed through the dimensionless velocity ratio, r:

$$r = \left|\vec{U}_P\right| / \left|\vec{U}\right| \qquad (4)$$

or the parallel (to the wall) component $U_P = \{Ux, Uy, 0\}$of the velocity magnitude divided by the velocity magnitude $U = \{Ux, Uy, Uz\}$. This definition of r is superior to others because it results in r being zero for perpendicular impacts. This results in $f(\psi,0)= 1$, and a uniform distribution of fingers, as would be expected. The model should extrapolate to shallow angle impacts without serious problems. At this point, we hypothesize that the functions f_1 and f_2 are only functionally related to r. In Part 1 of this series, it appears that there could be a pressure dependence that should also be considered for more general applicability. Insufficient data exist presently to construct a model that includes a pressure term.

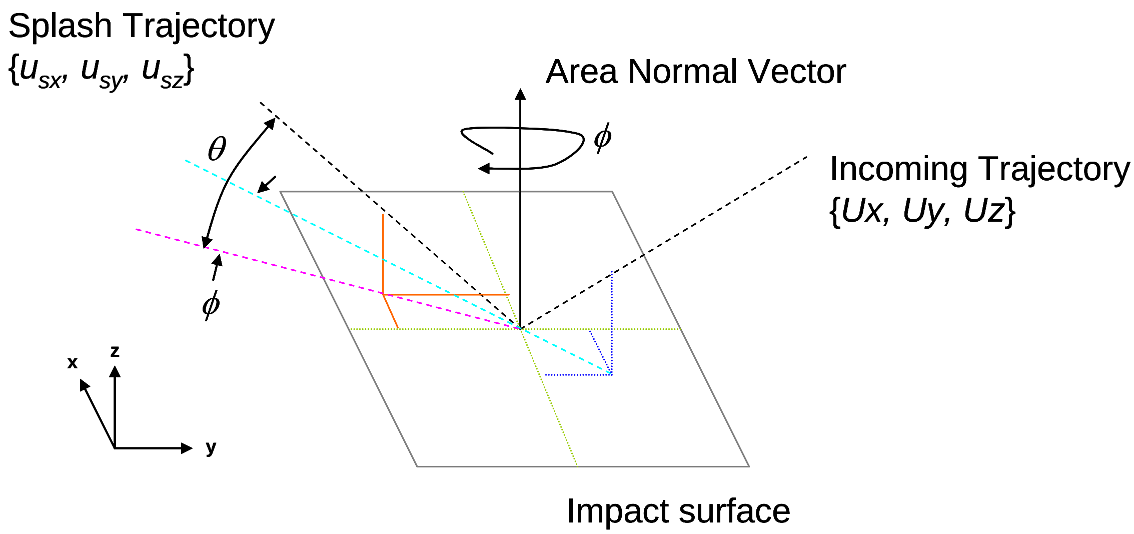

Figure 2. An illustration of impact splash variables for a single splash trajectory with velocity components illustrated with respect to a surface normal coordinate plane.

Using a ratio relationship, the angle ϕ corresponding to finger number N can be calculated by solving a distribution function relation:

$$\frac{N}{N_f} = \frac{\int_0^{\phi} \frac{1}{2} f(\psi, r) d\psi}{\int_0^{2\pi} \frac{1}{2} f(\psi, r) d\psi} \qquad (5)$$

This leaves two constants A and B in (2) and (3) that must be found that fit all of the data. The degree to which two constants are able to fit the data suggests the adequacy of the model form that has been selected.

The finger counts are non-dimensionalized by dividing a sectional count by the total finger count. These ratios should always be between zero and one. A fitting algorithm is used to find the value of the constants *A* and *B* that best fit the dimensionless data. We down-select from our dataset including only drop impacts with significant fingers continuous around the circumference of the spreading liquid and clean impacts. Subtleties such as minor localized instabilities that occur in transitional impacts are therefore not well represented in our data. We believe this limits the accuracy to the more energetic impacts (i.e. larger Weber numbers).

Results

Because (1) has a closed form solution and a known derivative, it is possible to construct in a spreadsheet a comparison between model and experiment and express the error in terms of summed least squares. A gradient reduction algorithm is used to optimize the reduction of the least squares and provide a fit for the missing fit constants A and B. Two empirical fit constants, *A* and *B*, have been found to reproduce a wide range of observations with:

$$A = 0.297 \qquad (6)$$
$$B = 0.723 \qquad (7)$$

These fit constants vary if portions of the calibration data are omitted, *A* values ranging in the 0.25 to 0.30 range and *B* values ranging between about 0.7 and 0.9. This is an expected range for future evaluations with additional data. The *A* constant affects the side distribution (QM), while the *B* constant affects the up-hill distribution of fingers. It is inferred from the magnitudes that the up-hill distribution is more affected by steepness of the angle.

Figure 3 shows the best fit comparison to the data nominally at atmospheric pressure from the experiments described in Part 1 of this series. A correlation analysis suggests a moderately good correlation ($R^2=0.9$). Most of the uncertainty lies in the smallest Weber number data where total finger counts are in the low double digits and fingering is near the transition point. The fit is much better for the large-scale data only, with $R^2=0.97$.

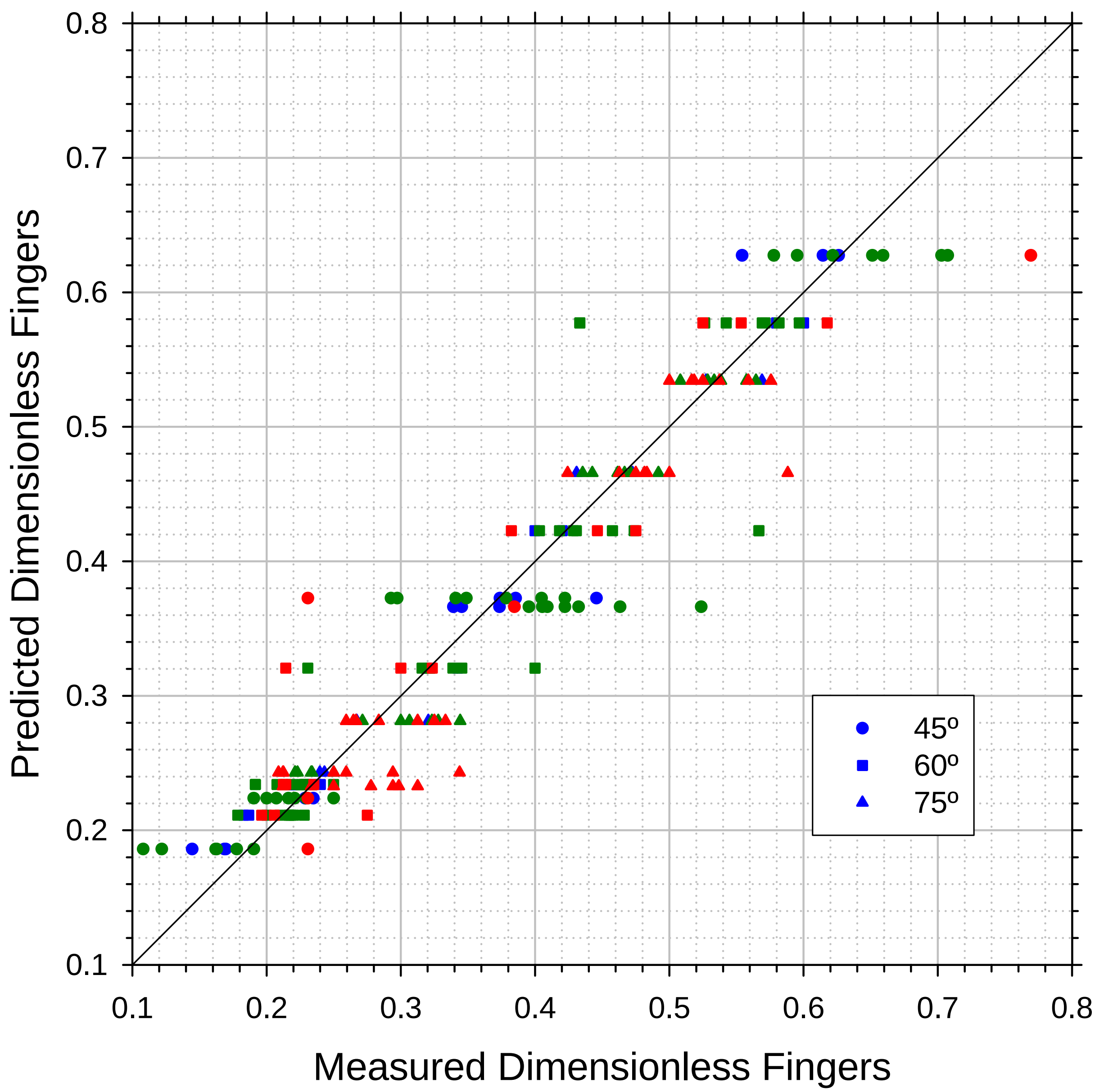

Figure 3. A comparison of predicted and measured dimensionless fingers using the present model and constants. UCR data are colored red, Sandia droptower data are blue, and Sandia 1 m drop data are dark green.

The dimensional data are not predicted directly by this model, so this model relies on an accurate model for the total number of fingers. The dimensional data may be compared by employing the model from Yoon et al. [4] for total number of fingers ($N_f=57log[We^*]-97$, where We^* employs only the perpendicular component of the total velocity vector). This comparison is found in Figure 4. Points are distinctive to illustrate the data source. This comparison suggests reasonable agreement between the model and the data.

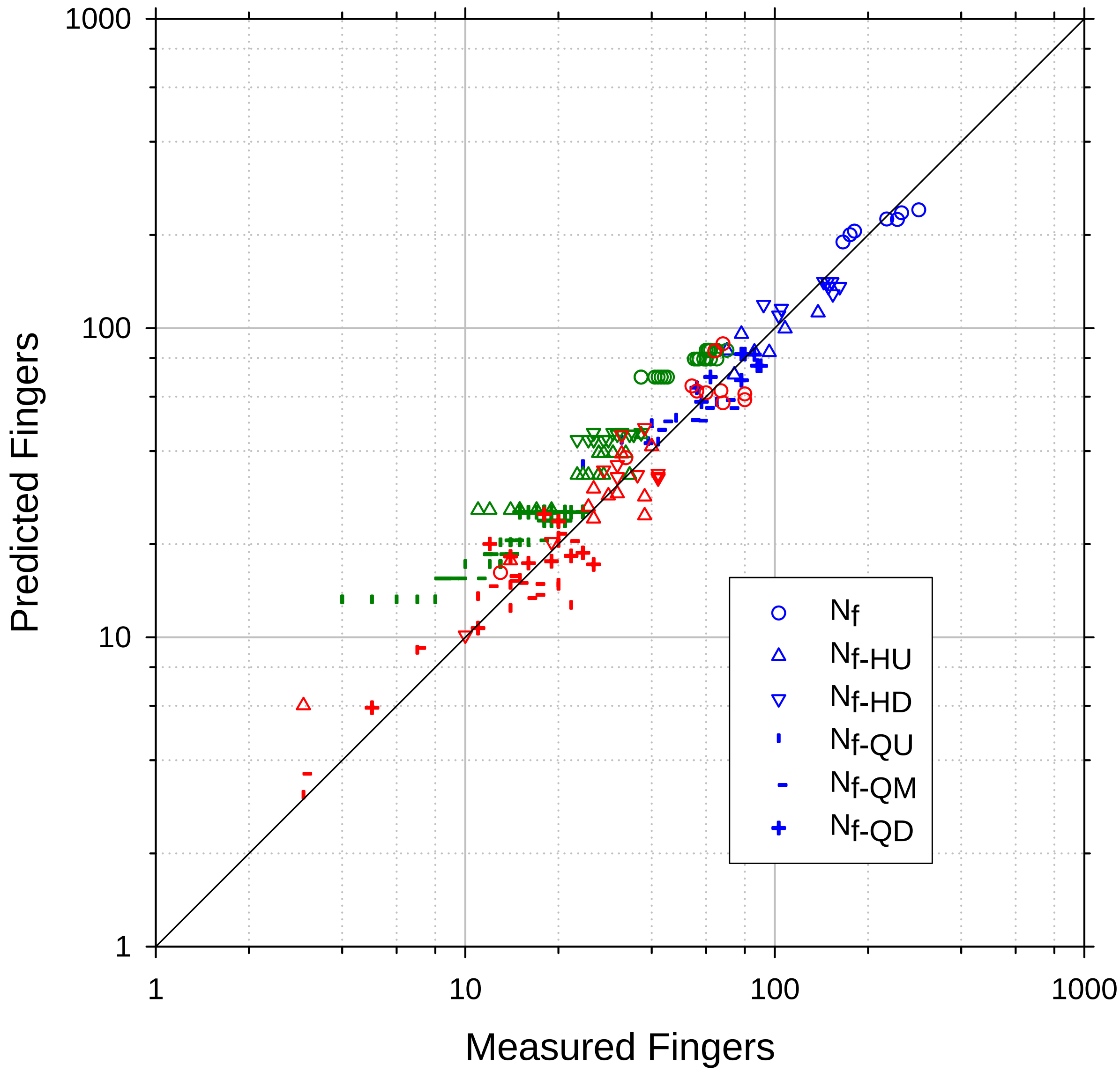

Figure 4. A comparison of predicted and measured fingers using the present model and constants. UCR data are colored red, Sandia droptower data are blue, and Sandia 1 m drop data are dark green.

Discussion

There appears to be a modest shift between the UCR data and the Sandia data for low finger counts in Figure 4. This appears to originate in a disagreement with total finger count, and perpetuates down to the fractionated finger counts. A correlation analysis still suggests $R^2 > 0.9$ between the prediction and the model.

This model represents what we believe to be the first attempt to compose a model for the distribution of fingers on oblique impact of a liquid drop that is based on measured data. Based on the data presently evaluated, the model performs well for water impacts in the range of velocities, diameters, and angles that have been studied. Its accuracy is also limited to single drop impacts on dry surfaces. More data will be necessary to evaluate whether the distribution is sensitive to other variations like the presence of a boundary layer, atmospheric pressure, surface roughness, viscosity of the liquid, and surface tension of the liquid.

The relationship between this model and the desire to model transportation accidents may not be obvious. Our interest in fingers is indirect. Fingers are the most obvious expression and easily measurable of the angular dependencies from oblique impacts. We believe that the mass distribution from such impacts is also angularly dependent, and we believe the dependency to be proportional to some extent to the finger distributions. Employing a similarity assumption, this model may then be employed as a mass distribution function that will respect the preferential distribution of mass in the direction of travel for an oblique impact. This is expected to be important for the break-up of liquid parcels from an energetic impact of liquid where the fast moving and large parcels of liquid potentially interact with unyielding solid surfaces. Many such impacts may be present in complex impact scenarios. We employ Lagrangian/Eulerian transport codes with reactions to model fires. This model represents a component of the impact and splash capabilities that is implemented and under verification in the codes. An accurate model for impacting drops is expected to help correctly model the momentum of the fluids in the system, and also affect the predicted atomization and evaporation of the fuel.

Nomenclature

		Greek		Subscripts	
A	Fit parameter for function f_1	ϕ	Angle of emerging finger	f	Finger
B	Fit parameter for function f_2	θ	Angle of lift of a drop	HD	Half down
f	An empirical function	ψ.	Angle	HU	Half up
N	Finger number			P	Parallel component
N_f	Number of fingers			QD	Quarter down
r	Dimensionless velocity			QM	Middle quarter
R^2	Coefficient of determination			QU	Quarter up
U	Velocity			x	Cartesian x
We^*	The Weber number			y	Cartesian y
				z	Cartesian z

Conclusions

We present a new model for the distribution of fingers around an impacting drop that respects the variations found between normal and oblique impacts. The model is shown to yield excellent accuracy for a wide range of drop impacts with significant fingering. It is also designed to behave well under extrapolative conditions. The impact finger distribution function is thought to be broadly applicable to impacting drop scenarios, but requires additional validation. It has been tested for water with various angles, drop diameters, and velocities. The oblique fingering model is a key component to the development and implementation of a new drop impact model in a Lagrangian/Eulerian coupled framework that is believed to be an improvement to existing models. The assumption that the fingering and mass distribution are related is key to the model.

Acknowledgements

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract No. DE-AC04-94AL85000.

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